

614-STATISTICS
(FINAL)

1. If n is an integer and $A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$, then find A^n .

(A) $\begin{bmatrix} 4n & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 4n & 0 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4n \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 4n & 1 \end{bmatrix}$

2. If P is an orthogonal matrix, then what is P^{-1} ?

(A) P

(B) P^T

(C) $-P$

(D) $-P^T$

3. Which one of the following conditions makes the expansion of $(2-3x)^{\frac{-11}{2}}$ valid?

(A) $x < \frac{2}{3}$

(B) $x \leq \frac{2}{3}$

(C) $|x| \leq \frac{2}{3}$

(D) $|x| < \frac{2}{3}$

4. What is the value of $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{3n}$?

- (A) e
- (B) e^3
- (C) e^{-3}
- (D) e^{-1}

5. What is the value of $\lim_{x \rightarrow a} \frac{x^{\frac{5}{8}} - a^{\frac{5}{8}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}}$?

- (A) $\frac{15}{8} a^{\frac{7}{24}}$
- (B) $\frac{8}{15} a^{\frac{7}{24}}$
- (C) $\frac{7}{24} a^{\frac{15}{8}}$
- (D) $\frac{7}{24} a^{\frac{8}{15}}$

6. What is the value of $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$?

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2

7. If $y = \sin^2 2x$, then find $\frac{dy}{dx}$.

- (A) $4 \sin 2x$
- (B) $2 \sin 2x \cos 2x$
- (C) $2 \cos 2x$
- (D) $2 \sin 4x$

8. If $y = 5^x$, then determine $\frac{dy}{dx}$.
- (A) $x5^{x-1}$
(B) $5x^4$
(C) $5\log_e 5$
(D) $5^x \log_e 5$
9. Find the value of $\lim_{x \rightarrow 0} \tan x \log x$.
- (A) 1
(B) 0
(C) -1
(D) Limit does not exist
10. Compute the slope of the tangent to the curve $y = \frac{6x}{x^2 - 1}$ at (2, 4).
- (A) $\frac{3}{10}$
(B) $-\frac{3}{10}$
(C) $\frac{10}{3}$
(D) $-\frac{10}{3}$
11. Find $\frac{\partial w}{\partial r}$, if $w = x^2 + y^2$, $x = r + e^s$ and $y = \log_e s$.
- (A) $r + e^s$
(B) $2r + 2e^s$
(C) e^s
(D) $r + 2e^s$

12. If $x = r \cos \vartheta$ and $y = r \sin \vartheta$, then find $\frac{\partial(x, y)}{\partial(r, \vartheta)}$.

- (A) r
- (B) $-r$
- (C) $r^2 \cos^2 \vartheta$
- (D) $r^2 \sin^2 \vartheta$

13. Evaluate $\int \tan^2 x dx$, (ignoring the constant of integration).

- (A) $\sec^2 x - x$
- (B) $\sec^2 x + x$
- (C) $\tan x - x$
- (D) $\tan x + x$

14. Find the value of $\int \frac{dx}{4+9x^2}$

- (A) $\frac{1}{9} \tan^{-1} \left(\frac{x}{2} \right)$
- (B) $\frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right)$
- (C) $\frac{1}{9} \sinh^{-1} \left(\frac{x}{2} \right)$
- (D) $\frac{1}{9} \sinh^{-1} \left(\frac{3x}{2} \right)$

15. Determine the value of $\int x e^x dx$

- (A) $x e^x$
- (B) $e^x - 1$
- (C) $x e^x - 1$
- (D) $e^x(x-1)$

16. Find the value of $\int_0^a \int_0^b (x^2 + y^2) dx dy$

(A) $ab(a^2 + b^2)$

(B) $\frac{ab}{3}(a^2 + b^2)$

(C) $\frac{a^2 + b^2}{3}$

(D) $a^2 + b^2$

17. What is the value of $\Gamma\left(\frac{1}{2}\right)$?

(A) π

(B) $\sqrt{\pi}$

(C) $\frac{1}{\pi}$

(D) $\frac{1}{2}$

18. Calculate the value of $\beta(2,5)$.

(A) $\frac{1}{30}$

(B) $\frac{1}{4}$

(C) $\frac{1}{1512}$

(D) $\frac{1}{120}$

19. Find the value of $\int_0^1 x^2(1-x)^3 dx$.

(A) $\frac{1}{30}$

(B) $\frac{1}{10}$

(C) $\frac{1}{35}$

(D) $\frac{1}{60}$

20. Compute the value of $\int_0^{\infty} x^3 e^{-2x} dx$.

(A) $\frac{1}{4}$

(B) $\frac{3}{8}$

(C) $\frac{3}{4}$

(D) $\frac{1}{9}$

21. Calculate the value of $\int_0^{\infty} e^{-x^2} dx$.

(A) $\frac{\sqrt{\pi}}{2}$

(B) $\sqrt{\frac{\pi}{2}}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{\sqrt{2}}$

22. Under which one of the following conditions $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots$ hold?
- (A) $-1 < x < 1$
 - (B) $x < 1$
 - (C) $x > 1$
 - (D) $-1 \leq x \leq 1$
23. If α and β are the roots of the equation $x^2 + px + q = 0$, then which one of the following determines the value of $\alpha^2 + \beta^2$?
- (A) $p^2 + 2q$
 - (B) $p^2 - 2q$
 - (C) $2p^2 - q$
 - (D) $2p^2 + q$
24. If Δ and E are finite difference operators defined as $\Delta y_x = y_{x+1} - y_x$ and $E y_x = y_{x+1}$, then which one of the following is true?
- (A) $E + \Delta = -1$
 - (B) $E + \Delta = 1$
 - (C) $\Delta + E = 0$
 - (D) $E - \Delta = 1$
25. Which one of the scales of measurement has unique origin?
- (A) Nominal
 - (B) Ordinal
 - (C) Interval
 - (D) Ratio
26. Temperature is a scale of measurement.
- (A) Nominal
 - (B) Ordinal
 - (C) Interval
 - (D) Ratio

27. Which one of the following can be used to present a frequency distribution visually?
- (A) Simple Bar chart
 - (B) Pie chart
 - (C) Histogram
 - (D) Subdivided Bar chart
28. Arithmetic mean of a set of 7 observations is 15 and of a set of 10 observations is 8, what is the mean of combined sample?
- (A) 10.80
 - (B) 10.88
 - (C) 15
 - (D) 1.35
29. A car travels 200 km at a speed of 80 km/hr and another car travels 350 km at a speed of 100 km/hr. What is the average speed for the whole journey?
- (A) 90.67
 - (B) 100
 - (C) 91.67
 - (D) 80.67
30. Which of the following measures of location is suitable to calculate the rate of population growth?
- (A) Arithmetic Mean
 - (B) Geometric Mean
 - (C) Harmonic Mean
 - (D) Mode
31. If the coefficient of variation for a data related to salary of 600 employees with standard deviation Rs. 10 is 5, then what is the average salary of employees?
- (A) Rs. 200
 - (B) Rs. 10
 - (C) Rs. 500
 - (D) Rs. 20

32. If the standard deviation of mesokurtic distribution is 5, then what is the value of the fourth order central moment?
- (A) 75
 - (B) 1875
 - (C) 43.30
 - (D) 8.66
33. Mean and standard deviation of 50 observations are respectively 30 and 4. If each observation is added with 6, what are the values of the mean and standard deviation of the new set of observations?
- (A) 36, 10
 - (B) 36, 4
 - (C) 30, 10
 - (D) 30, 4
34. Which of the following **cannot** be easily approximated by a Box-Whisker plot?
- (A) Range
 - (B) Inter Quartile Range
 - (C) Second Quartile
 - (D) Standard Deviation
35. Which one of the following is a meaningful measure of location for nominal data?
- (A) Arithmetic Mean
 - (B) Geometric Mean
 - (C) Median
 - (D) Mode
36. If the sample size increases, which one of the following statements hold?
- (A) Both sampling error and non-sampling error decrease
 - (B) Sampling error decreases and non-sampling error increases
 - (C) Both sampling error and non-sampling error increase
 - (D) Sampling error increases and non-sampling error decreases
37. Which one of the following is **NOT** a frequency data?
- (A) Income distribution data of India
 - (B) Age composition data of India
 - (C) Annual production of steel in India during 1990-2017
 - (D) Distribution of marks obtained by students of a class in Mathematics

38. Which one of the following can be found from ogive?
- (A) Arithmetic Mean
 - (B) Median
 - (C) Mode
 - (D) Geometric Mean
39. Which one of the following relations hold for a positively skewed distribution?
- (A) Mean > Median > Mode
 - (B) Mean < Median < Mode
 - (C) Mean > Mode > Median
 - (D) Mean < Mode < Median
40. If the frequency of the i^{th} observation x_i is f_i and total frequency is N , which one of the following formula can be used to compute the mean deviation about the sample mean \bar{x} ?

(A) $\frac{2}{N} \left(\bar{x} \sum_{x_i > \bar{x}} f_i - \sum_{x_i > \bar{x}} f_i x_i \right)$

(B) $\frac{2}{N} \left(\sum_{x_i < \bar{x}} f_i - \sum_{x_i > \bar{x}} f_i x_i \right)$

(C) $\frac{2}{N} \left(\bar{x} \sum_{x_i > \bar{x}} f_i - \sum_{x_i < \bar{x}} x_i \right)$

(D) $\frac{2}{N} \left(\bar{x} \sum_{x_i < \bar{x}} f_i - \sum_{x_i < \bar{x}} f_i x_i \right)$

41. If two random variables X and Y have the standard deviations respectively σ_X and σ_Y with correlation co-efficient r , then which one of the following can be used to determine the angle between two regression lines?

(A) $\theta = \tan^{-1} \left\{ \frac{1-r^2}{|r|} \left(\frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2} \right) \right\}$

(B) $\theta = \tan^{-1} \left\{ \frac{1-r^2}{|r|} \left(\frac{\sigma_X^2 + \sigma_Y^2}{\sigma_X \sigma_Y} \right) \right\}$

(C) $\theta = \tan^{-1} \left\{ \frac{(1-r^2)}{|r|} \times \frac{\sigma_Y}{\sigma_X} \right\}$

(D) $\theta = \tan^{-1} \left\{ \frac{(1-r^2)}{|r|} \times \frac{\sigma_X}{\sigma_Y} \right\}$

42. If two random variables X and Y have the following joint probability density function

$$f(x, y) = \begin{cases} xe^{-x(y+1)}, & x \geq 0, y \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$$

then which one of the following represents the regression line of Y on X ?

(A) $Y = X$

(B) $Y = X^2$

(C) $Y = \frac{1}{X^2}$

(D) $Y = \frac{1}{X}$

43. Two random variables X and Y are connected by the equation $aX + bY + c = 0$, then which one of the statements holds?

(A) the correlation coefficient between them is +1, if signs of a and b are alike and -1 if the signs of a and b are different

(B) the correlation coefficient between them is -1, if signs of a and b are alike and +1 if the signs of a and b are different

(C) the correlation coefficient between them is +1, if $a, b > 0$ and $c < 0$

(D) the correlation coefficient between them is -1, if $a, c > 0$ and $b < 0$

44. If all the pairwise correlation coefficients in a three-variable data is ρ , then what is the formula for computing the partial correlation coefficient?

(A) $\frac{\rho}{1+\rho}$

(B) $\frac{\rho^2}{1+\rho^2}$

(C) $\frac{1-\rho}{\rho}$

(D) ρ

45. In dichotomous classification of n attributes, what is the total number of class frequencies?

(A) n^2

(B) 2^n

(C) $n+2$

(D) $2n$

46. If $P(A) = \frac{2}{3}$ and $P(A^C | B^C) = \frac{1}{2}$, what is the value of $P(B)$?

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

47. A family, selected at random from the set of all families living in a city with two children, is found to have a girl child. What is the probability that other child of the family is also a girl?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{3}{4}$

(D) $\frac{1}{2}$

48. Two cards are drawn from a well shuffled ordinary pack of 52 cards. What is the probability that both are kings, if the first card is not replaced?

(A) $\frac{1}{13}$

(B) $\frac{3}{51}$

(C) $\frac{1}{221}$

(D) $\frac{1}{663}$

49. If A and B are two mutually exclusive events with $P(A \cup B) \neq 0$, then which one of the following can be used to compute $P(A | A \cup B)$?

(A) $\frac{P(A)}{P(A \cap B)}$

(B) $\frac{P(B)}{P(A \cap B)}$

(C) $\frac{P(A)}{P(A) + P(B)}$

(D) $\frac{P(B)}{P(A) + P(B)}$

50. Let B_1, B_2, \dots, B_k be mutually exclusive events such that $P(B_j) \neq 0$, $j = 1, 2, \dots, k$ are known and they form a partition of the sample space. If A is an event contained in $\bigcup_{j=1}^k B_j$ with $P(A) > 0$ and $P(B_j | A)$ is given for all j , what is the probability of B_1 given A ?

(A)
$$\frac{P(B_1)P(A|B_1)}{\sum_{j=1}^k P(B_j)P(A|B_j)}$$

(B)
$$\frac{P(A)P(B_1|A)}{\sum_{j=1}^k P(B_j)P(A|B_j)}$$

(C)
$$\frac{P(A|B_1)}{\sum_{j=1}^k P(B_j)P(A|B_j)}$$

(D)
$$\frac{P(A|B_1)}{\sum_{j=1}^k P(A)P(A|B_j)}$$

51. The probability density function of a random variable X is given

by
$$f(x) = \begin{cases} \frac{x^2}{81}; & -3 < x < 6 \\ 0 & ; \text{otherwise} \end{cases}$$

If $U = \left(\frac{1}{3}\right)(12 - X)$, then what is the value of $P(2 < U < 5)$?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

52. What is the average distance of two points taken at random on the circumference of a circle with radius ' r '?

(A) $\frac{r}{\pi}$

(B) $\frac{2r}{\pi}$

(C) $\frac{3r}{\pi}$

(D) $\frac{4r}{\pi}$

53. If X and Y are two random variables, which one of the following is correct?

(A) $Var(X) = E[Var(X|Y)] + Var[E(X|Y)]$

(B) $Var(X) = E[Var(Y|X)] + Var[E(Y|X)]$

(C) $Var(X) = E[Var(X|Y)] - Var[E(X|Y)]$

(D) $Var(X) = E[Var(X|Y)]$

54. A random variable X has the probability density function $f(x) = \alpha x e^{-\beta^2 x^2}$, if $x > 0$,

$\alpha > 0$, $\beta > 0$. If $E(X) = \frac{\sqrt{\pi}}{2}$, then what are the values of α and β ?

(A) 1 and 2

(B) 2 and 1

(C) 1 and π

(D) π and 1

55. For a random variable X , $P[X = 0] = 1 - \frac{1}{k^2}$ and $P[X = +1] = P[X = -1] = \frac{1}{2k^2}$ where k is a constant greater than 1. Then which of the following is correct, if σ is the standard deviation of X ?

- (A) $P[|x| \geq k\sigma] < \frac{1}{k^2}$
(B) $P[|x| \geq k\sigma] > 1 - \frac{1}{k^2}$
(C) $P[|x| \geq k\sigma] = \frac{1}{k^2}$
(D) $P[|x-1| \geq k\sigma] = \frac{1}{k^2}$

56. Let (X, Y) be jointly distributed with the density function

$$f(x, y) = \begin{cases} e^{-y}; & 0 < x < y < \infty \\ 0 & ; \text{otherwise} \end{cases}$$

Consider the following:

- (i) $E(Y) = 2$
(ii) $E(X | Y) = \frac{y}{2}$, $0 < y < \infty$

Which one of the above statements is correct?

- (A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

57. If two independent variables X_1 and X_2 have Poisson distribution with means m_1 and m_2 respectively, what is the moment generating function of $X_1 - X_2$?

- (A) $e^{(m_1 - m_2)t}$
(B) $e^{m_1 e^t + m_2 e^{-t} - m_1 - m_2}$
(C) $e^{m_1 e^t + m_2 e^{-t} - 1}$
(D) $e^{m_1 e^t - m_2 e^{-t} - 1}$

58. If the probability mass function of a random variable X is given by $P(X = x) = e^{-t} (1 - e^{-t})^x$, for $x = 0, 1, 2, \dots$ and t is a positive real number, what is the mean of X ?
- (A) e^t
(B) e^{-t}
(C) $e^t - 1$
(D) $1 + e^t$
59. Which one of the following statements is correct?
- (A) Sum of two independent Bernoulli random variables is a Bernoulli random variable
(B) Sum of any two independent binomial random variables is always a binomial random variable
(C) Sum of two independent and identical Bernoulli random variables is a binomial random variable
(D) Sum of two independent binomial random variables is a negative random variable
60. If X follows normal distribution with mean 2 and variance 25, then what is the distribution of $Y = \frac{1}{2} \left(\frac{X - 2}{5} \right)^2$?
- (A) Exponential with mean 4
(B) Chi-square with 2 degrees of freedom
(C) Gamma distribution with parameter $\left(\frac{1}{2}, 1 \right)$
(D) Normal distribution

61. Buses arrive at a specified bus stop in 15-minute interval from 7 a.m., everyday. If a passenger arrives at the stop at a time that uniformly distributed between 7 a.m. and 7.30 a.m., what is the probability that the passenger waits less than 5 minutes for a bus?

- (A) $\frac{1}{6}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{1}{10}$

62. Let X_1, X_2, \dots, X_n be a random sample from the normal distribution with mean μ and variance σ^2 . Define $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$, $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$. Consider the following Statements.

- (i) \bar{X} and S^2 are independent
(ii) S^2 is a biased estimator of σ^2

Which one of the following is true?

- (A) (i) only
(B) (ii) only
(C) Both (i) and (ii)
(D) Neither (i) nor (ii)

63. If X_1, X_2, \dots, X_n are independent standard normal random variables and $S_n = \sum_{i=1}^n X_i^2$.

Which one of the following statements is correct?

- (A) The distribution of S_n is F
(B) The distribution of S_n is χ_n^2
(C) The distribution of S_n is Normal $(0, n)$
(D) The distribution of S_n is t_{n-1}

64. Let X_1, X_2, \dots, X_n be a random sample from the exponential distribution with mean $\frac{1}{\lambda}$.
What is the maximum likelihood estimator of the median of the distribution?

- (A) $\frac{\bar{X}}{(\ln 2)}$
(B) $\bar{X} \ln 2$
(C) $\frac{\ln 2}{\bar{X}}$
(D) $\ln(2\bar{X})$

65. Let X_1, X_2, \dots, X_n be a random sample from Binomial $(1, \theta)$ and $T = X_1 + X_2 + \dots + X_n$. Then, which one of the following is an unbiased estimator of θ^2 ?

- (A) $\frac{T(T+1)}{n(n+1)}$
(B) $\frac{T(T+1)}{n(n-1)}$
(C) $\frac{T(T-1)}{n(n+1)}$
(D) $\frac{T(T-1)}{n(n-1)}$

66. What is the minimum variance bound unbiased estimator of θ^2 based on a random sample of size 'n' from the Normal $(0, \theta^2)$ distribution?

- (A) $\left(\frac{1}{n} \sum_{i=1}^n X_i^2\right)^{-1}$
(B) $\frac{1}{n} \sum_{i=1}^n X_i^2$
(C) $\frac{1}{n} \left(\sum_{i=1}^n X_i\right)^2$
(D) $\frac{1}{n^2} \sum_{i=1}^n X_i^2$

67. Let the random variable $X \sim \text{Uniform}(5, 5 + \mu)$. Based on a random sample of size one, say X_1 , which one of the following is an unbiased estimator of μ^2 ?

(A) $3(X_1 - 5)^2$

(B) $\frac{X_1^2 - 5}{12}$

(C) $3(X_1 + 5)^2$

(D) $\frac{X_1^2 + 5}{12}$

68. Let X_1, X_2 and X_3 be independent random variables with $X_k (k=1, 2, 3)$ having the probability density function $f_k(x) = \begin{cases} k\theta e^{-k\theta x}; & x > 0 \\ 0 & ; \text{otherwise} \end{cases}$, where $\theta > 0$. Then which one of the following is a sufficient statistic for θ ?

(A) $X_1 + X_2 + X_3$

(B) $3X_1 - 2X_2 - X_3$

(C) $X_1 + X_2 - 3X_3$

(D) $X_1 + 2X_2 + 3X_3$

69. Let X_1, X_2, \dots, X_n be a random sample of size 'n' from the Normal $(\mu, 16)$ population and z_p denote the $100p^{\text{th}}$ percentile of the standard normal distribution. How many observations would have been drawn for constructing the 95% confidence interval $[\bar{x} - 0.98, \bar{x} + 0.98]$ for μ ?

(A) $4.0816z_{0.975}$

(B) $4.0816z_{0.025}^2$

(C) $16.66z_{0.025}$

(D) $16.66z_{0.975}^2$

70. Let X_1, X_2, \dots, X_n be a random sample Bernoulli population with parameter p and \bar{X} be the sample mean. Which of the following is incorrect?
- (A) $\bar{X}(1 - \bar{X})$ is a consistent estimator of $p(1 - p)$
 - (B) $\bar{X}(1 - \bar{X})$ is an unbiased estimator of $p(1 - p)$
 - (C) \bar{X} is a consistent estimator of p
 - (D) \bar{X} is an unbiased estimator of p
71. In testing of the null hypothesis H_0 against the alternative hypothesis H_1 , let x denote the sample and W denotes the critical region. Which of the following is defined as power of the test?
- (A) $P\{x \in W \mid H_0 \text{ is true}\}$
 - (B) $P\{x \notin W \mid H_0 \text{ is true}\}$
 - (C) $P\{x \in W \mid H_0 \text{ is false}\}$
 - (D) $P\{x \notin W \mid H_0 \text{ is false}\}$
72. A coin is tossed 4 times and 'p' is the probability of getting head in a single trial. Let S be the number of heads obtained. It is decided to test $H_0: p = \frac{1}{2}$ against $H_1: p \neq \frac{1}{2}$, using the decision rule: Reject H_0 if $S = 0$ or 4. What are the probabilities of Type II error (β) when $p = \frac{3}{4}$ and Type I error (α)?
- (A) $\alpha = \frac{1}{4}, \beta = \frac{87}{128}$
 - (B) $\alpha = \frac{1}{8}, \beta = \frac{87}{128}$
 - (C) $\alpha = \frac{1}{8}, \beta = \frac{41}{256}$
 - (D) $\alpha = \frac{1}{4}, \beta = \frac{41}{256}$

73. Let X_1, X_2, \dots, X_n be a random sample of n observations drawn from a population having the probability density function $f(x|\theta) = \begin{cases} (\theta+1)x^\theta, & \text{if } 0 < x < 1 \text{ and } \theta = 1, 2 \\ 0 & \text{, otherwise} \end{cases}$.

Which one of the following statistics determines the most powerful test for testing $H_0: \theta = 1$ against $H_1: \theta = 2$?

- (A) $\sum_{i=1}^n X_i$
 (B) $\prod_{i=1}^n X_i$
 (C) $\sum_{i=1}^n \frac{1}{X_i}$
 (D) $\frac{1}{\log\left(\prod_{i=1}^n X_i\right)}$

74. In testing statistical hypotheses, the likelihood ratio test statistic is necessarily a function of which one of the following?

- (A) Moment estimators
 (B) Maximum likelihood estimators
 (C) Least squares estimators
 (D) Minimum χ^2 estimators

75. If r and ρ denote respectively the sample and population correlation co-efficient, then which one of the following is the test statistic for testing $H_0: \rho = 0$?

- (A) $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$
 (B) $\frac{r^2\sqrt{n-2}}{\sqrt{1-r}}$
 (C) $\frac{r\sqrt{n-1}}{\sqrt{1-r^2}}$
 (D) $\frac{r\sqrt{n-2}}{\sqrt{1-r}}$

76. Which one of the following relationship represents the Neyman's allocation for fixed total sample size?
- (A) $n_i \propto N_i$
 - (B) $n_i \propto N_i S_i$
 - (C) $n_i \propto \frac{N_i S_i}{\sqrt{c_i}}$
 - (D) None of the above
77. Which one of the following is true for simple random sampling without replacement?
- (A) Sample mean is an unbiased estimator for population mean
 - (B) $\frac{1}{n-1} \sum (y_i - \bar{y})^2$ is an unbiased estimator of $\frac{1}{N-1} \sum (Y_i - \bar{Y})^2$
 - (C) Possible number of samples is $\binom{N}{n}$
 - (D) All of the above
78. Which one of the following kinds of errors is **NOT** discussed for complete enumeration method?
- (A) Sampling error
 - (B) Non-sampling error
 - (C) Response error
 - (D) Measurement error

79. Let S^2 and s^2 denote respectively the population and sample mean square. Which one of the following is the standard error of the mean of a random sample of size n drawn under simple random sampling without replacement from a population of N observations?

(A) $\sqrt{\frac{N-n}{nN}}s^2$

(B) $\sqrt{\frac{N-n}{N}}s$

(C) $\sqrt{\frac{N-n}{nN}}S$

(D) $\sqrt{\frac{N-n}{nN}}S^2$

80. A random sample of 10 observations has to be drawn under systematic sampling scheme from a population consisting of 500 units. If position of the first observation is eleven, which one of the following is the position of the seventh observation in the sample?

- (A) 261
- (B) 311
- (C) 361
- (D) 411

81. Let variance of the mean of a random sample drawn under simple random sampling without replacement, stratified sampling with proportional allocation and stratified sampling with Neyman's allocation be respectively $Var(\bar{y}_{SRS})$, $Var(\bar{y}_P)$ and $Var(\bar{y}_O)$. Which one of the following holds?

- (A) $Var(\bar{y}_{SRS}) \geq Var(\bar{y}_P) \geq Var(\bar{y}_O)$
- (B) $Var(\bar{y}_O) \geq Var(\bar{y}_P) \geq Var(\bar{y}_{SRS})$
- (C) $Var(\bar{y}_{SRS}) \geq Var(\bar{y}_O) \geq Var(\bar{y}_P)$
- (D) $Var(\bar{y}_P) \geq Var(\bar{y}_{SRS}) \geq Var(\bar{y}_O)$

82. Which one of the following factors requires selection of a sample with large size?

- (A) Heterogeneity of sampling units
- (B) High precision of the estimate
- (C) Both heterogeneity of sampling units and high precision of the estimate
- (D) Neither heterogeneity of sampling units nor high precision of the estimate

83. If an experiment is replicated r times and σ is the population standard deviation, which one of the following can be used for computing the precision?

- (A) $\frac{r}{\sigma^2}$
- (B) $\frac{r}{\sigma}$
- (C) $\frac{\sigma^2}{r}$
- (D) $\frac{\sigma}{r}$

84. Which one of the following distributions is used as the sampling distribution of the test statistic in analysis of variance?

- (A) Student's t
- (B) F
- (C) χ^2
- (D) Normal

85. What does the term $\sum_i \sum_j (y_{ij} - \bar{y}_i)^2$ represent in the analysis of one-way classified data?

- (A) Variance
- (B) Total sum of squares
- (C) Sum of squares due to treatments
- (D) Error sum of squares

86. Let α_i be the effect due to i^{th} factor, β_j be the effect due to j^{th} factor and γ_{ij} be the interaction effect in a two-way classified data with one observation per cell. Under which one of the following assumptions, the linear model $y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ij}$ is used for analyzing the data?
- (A) $\alpha_i = 0$
 - (B) $\beta_j = 0$
 - (C) $\gamma_{ij} = 0$
 - (D) $\gamma_{ij} = 1$
87. What are the degrees of freedom associated with error sum of squares in the analysis of two-way classified data with one observation per cell?
- (A) $n - 1$
 - (B) $k - 1$
 - (C) $(n - 1)(k - 1)$
 - (D) $nk - 1$
88. Which one of the following is called the process of reducing the experimental error by dividing the relatively heterogeneous experimental area into homogeneous blocks?
- (A) Error
 - (B) Replication
 - (C) Randomization
 - (D) Local control

89. Which one of the following formula is used to compute the standard error of the difference between any two treatment means of a randomized block design with 5 treatments and 3 blocks?

(A) $\left(\frac{3s_e^2}{5}\right)^{\frac{1}{2}}$

(B) $\left(\frac{3s_e^2}{2}\right)^{\frac{1}{2}}$

(C) $\left(\frac{5s_e^2}{3}\right)^{\frac{1}{2}}$

(D) $\left(\frac{2s_e^2}{3}\right)^{\frac{1}{2}}$

90. How many mutually orthogonal contrasts are there in a 2^3 factorial experiment?

- (A) 8
- (B) 4
- (C) 7
- (D) 9

91. Which one of the following systems provides national as well as state level reliable estimates of fertility and mortality?

- (A) Civil registration system
- (B) Census
- (C) Ad-hoc survey
- (D) Sample registration system

92. Which one of the following rates is used by demographers as an indicator of the quality of a population's healthcare?

- (A) Age-specific death rate
- (B) Age-specific birth rate
- (C) Crude death rate
- (D) Infant mortality rate

93. What are the reasons based on which R-charts are preferred over S-charts?
- (A) Sample range can be easily calculated
 - (B) R charts are economical
 - (C) Sample range and sample standard deviation fluctuate together in the case of small samples
 - (D) All of the above
94. Which one of the following formulae is used to compute the average outgoing quality (AOQ) in terms of the lot acceptance probability (P_a) and incoming lot quality (p)?
- (A) $AOQ = pP_a$
 - (B) $AOQ = (N - n)pP_a$
 - (C) $AOQ = \frac{N - n}{N} pP_a$
 - (D) $AOQ = \frac{N - n}{N} P_a$
95. Which one of the following curves can exhibit the proportion of submitted lots that will be accepted on the basis of sampling plan for each percentage of defective items in the lots under consideration?
- (A) Power curve
 - (B) ASN curve
 - (C) OC curve
 - (D) Gompertz curve
96. Which one of the following indices is the arithmetic mean of Laspeyre's and Paasche's index formulae?
- (A) Drobish-Bowley Index
 - (B) Fisher's Index
 - (C) Walsh Index
 - (D) Kelly's Index
97. Which one of the following methods can be used to compare two series of index numbers with different base periods?
- (A) Base shifting
 - (B) Splicing
 - (C) Deflating
 - (D) Link relatives

98. Which one of the following statements is/are correct?
- (i) Hungarian method is used to solve assignment problem as well as travelling salesman problem
 - (ii) Travelling salesman problem is one in which cost is the largest number along the principle diagonal
- (A) Both (i) and (ii) are correct
 - (B) Only (i) is correct
 - (C) Only (ii) is correct
 - (D) Both (i) and (ii) are incorrect
99. Find $\frac{d \cosh^{-1} x}{dx}$.
- (A) $\frac{1}{\sqrt{1+x^2}}$
 - (B) $\frac{1}{1+x^2}$
 - (C) $\frac{1}{\sqrt{x^2-1}}$, ($x > 1$)
 - (D) $\frac{1}{x^2-1}$, ($x \neq 1$)
100. A producer can sell x items per week at a price $P = 200 - 0.1x$ (in rupees) and the production cost per item is $y = 50x + 20000$ (in rupees). Which one of the following is the number of items yielding maximum profit?
- (A) 75
 - (B) 7500
 - (C) 10000
 - (D) 20000
101. Which one of the following is much affected by outliers?
- (A) Median
 - (B) Geometric Mean
 - (C) Range
 - (D) Inter-quartile Range

102. The coefficient of symmetry for the t -distribution with 7 degrees of freedom is
- (A) 0
 - (B) $\frac{1}{2}$
 - (C) 1
 - (D) 2
103. Let μ_i and σ_i^2 be respectively the mean and variance of the i^{th} ($i = 1, 2, \dots, k$) normal population. Also, let R represent the multiple correlation co-efficient. In which one of the following hypotheses testing problems, F -distribution cannot be applied?
- (A) $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$
 - (B) $H_0 : \sigma_1^2 = 25$ against $H_1 : \sigma_1^2 \neq 25$
 - (C) $H_0 : R^2 = 0$ against $H_1 : R^2 > 0$
 - (D) $H_0 : \mu_1 = \mu_2 = \dots = \mu_k$ against $H_1 : \mu_i \neq \mu_j$ for some $i \neq j$
104. Let c_1, c_2, \dots, c_k be constants and t_1, t_2, \dots, t_k denote the effects of k treatments. Which one of the following conditions specifies that the treatment combination $c_1 t_1 + c_2 t_2 + \dots + c_k t_k$ is a contrast?
- (A) $\sum_{i=1}^k t_i = 0$
 - (B) $\sum_{i=1}^k c_i = 0$
 - (C) $\sum_{i=1}^k c_i t_i = 1$
 - (D) $\sum_{i=1}^k c_i = 1$
105. When there are many items to be inspected and the testing is destructive, which one of the following is used?
- (A) Acceptance sampling
 - (B) 0% inspection
 - (C) 50% inspection
 - (D) 100% inspection

106. Nine customers arrive at a departmental store, on an average, every 5 minutes. The cashier serves, on an average, to ten customers in 5 minutes. Let arrival rate follow Poisson distribution and the service time follow exponential distribution.

What is the average number of customers in the system?

- (A) 9
- (B) 8
- (C) 16
- (D) 7

107. Which of the following functions is **NOT** convex?

- (A) $f(x) = x^2$
- (B) $f(x) = e^x$
- (C) $f(x) = \log_e x$
- (D) $f(x) = 7 - x$

108. The least value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ for positive x, y, z satisfying the conditions

$x + y + z = 9$ is

- (A) $\frac{15}{7}$
- (B) $\frac{1}{9}$
- (C) 3
- (D) 1

109. An unbiased coin is tossed up 4 times. The probability that tails turn up in 3 cases is

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{6}$

110. For a Poisson distribution, if $P(1) = P(3)$ then, what is the mean?
- (A) $\sqrt{2}$
 - (B) $\sqrt{3}$
 - (C) $\sqrt{6}$
 - (D) $\sqrt{7}$
111. A bag contains 6 pairs of socks. If 2 pairs of socks are selected at random with replacement then the number of possible samples is?
- (A) 6
 - (B) 12
 - (C) 36
 - (D) 216
112. A bag contains 80 chocolates. This bag has 4 different colors of chocolates in it. If all four colors of chocolates were equally likely to be put in the bag, what would be the expected number of chocolates of each color?
- (A) 12
 - (B) 11
 - (C) 20
 - (D) 9
113. Cumulative Frequency Curve is also called
- (A) Frequency Polygon
 - (B) Frequency Curve
 - (C) Histogram
 - (D) Ogive
114. In a Pie chart one can calculate the angles for each sector by the following formula
- (A) $\frac{\text{Component Part}}{\text{Total}} \times 100$
 - (B) $\frac{\text{Component Part}}{\text{Total}} \times \pi$
 - (C) $\frac{\text{Total}}{\text{Component Part}} \times 360^\circ$
 - (D) $\frac{\text{Component Part}}{\text{Total}} \times 360^\circ$

115. A curve that tails off to the right end is called
- (A) Negatively skewed
 - (B) Symmetrical
 - (C) Positively skewed
 - (D) J curve
116. The measure of central tendency is shown in a box plot diagram
- (A) Harmonic Mean
 - (B) Median
 - (C) Weighted Mean
 - (D) Geometric Mean
117. Which of the following statement is **FALSE**?
- (A) Ranked or ordinal data is presented with Pie chart
 - (B) In a positively skewed histogram, the mean is larger than the median
 - (C) A histogram can have more than one mode
 - (D) In stem and leaf display the individual values don't vanish
118. Find the number of all possible samples from a population containing 8 items from which 2 items are selected at random without replacement.
- (A) 56
 - (B) 28
 - (C) 66
 - (D) 38
119. In systematic sampling, population is 240 and selected sample size is 60 then sampling interval is
- (A) 240
 - (B) 60
 - (C) 4
 - (D) 0.25
120. Stratified Sample is used when the population is
- (A) Homogeneous
 - (B) Heterogeneous
 - (C) Vary large
 - (D) Too Small

121. An example of Probability Sampling is
- (A) Quota sampling
 - (B) Judgmental sampling
 - (C) Purposive sampling
 - (D) Lottery Method
122. The average growth of a certain variety of pine trees is 10.1 inches in three years. A biologist claims that a new variety will have greater three-year growth. A random sample of 25 of the new variety has an average three-year growth of 10.8 inches and a standard deviation of 2.1 inches. The appropriate null and alternative hypotheses to test the biologist's claim are
- (A) $H : \mu = 10.8$ against $H_a : \mu > 10.8$
 - (B) $H : \mu = 10.8$ against $H_a : \mu \neq 10.8$
 - (C) $H : \mu = 10.1$ against $H_a : \mu > 10.1$
 - (D) $H : \mu = 10.1$ against $H_a : \mu \neq 10.1$
123. In hypothesis testing, if type II error is represented by β , then the power of the test is
- (A) The probability of rejecting H_0 when H_1 is true
 - (B) The probability of failing to reject H_0 when H_1 is true
 - (C) The probability of failing to reject H_0 when H_0 is true
 - (D) The probability of rejecting H_0 when H_0 is true
124. In statistical hypothesis test of equality of means, such as $H_0 : \mu = 10$, if $\alpha = 5\%$
- (A) 95% of the time we will make an incorrect inference
 - (B) 5% of the time we will say that there is a real difference when there is no difference
 - (C) 5% of the time we will say that there is no real difference when there is a difference
 - (D) 95% of the time the null hypothesis will be correct
125. The Spearman rank correlation test requires that the
- (A) Data must be measured on the same scale
 - (B) Data should be of ordinal scale at least
 - (C) Data must be distribution at least approximately as a t-distribution
 - (D) Data must be from two independent samples

126. When the correlation coefficient r is close to one,
- (A) there is no relationship between the two variables
 - (B) there is a strong linear relationship between the two variables
 - (C) it is impossible to tell if there is a relationship between the two variables
 - (D) the slope of the regression line will be close to one
127. Let $V(T)$ denote the variance of an estimator T . Then, the mean squared error of T is equal to
- (A) $V(T) + \text{Bias}$
 - (B) $V(T) + (\text{Bias})^2$
 - (C) $V(T) - \text{Bias}$
 - (D) $V(T) - (\text{Bias})^2$
128. Analysis of variance is used for testing
- (A) Two population variances
 - (B) Two or more population means
 - (C) Two population means
 - (D) Two or more population variances
129. Consider k independent samples each containing n_1, n_2, \dots, n_k items such that $n_1 + n_2 + \dots + n_k = n$. In ANOVA we use F-distribution with degree of freedom
- (A) $k-1, n-k$
 - (B) $k-1, n-1$
 - (C) $k-n, n-k$
 - (D) $n-k, k-1$
130. In one-way ANOVA, given Sum of Squares due to Treatment (SST) = 2580, Sum of Squares due to Error (SSE) = 1656, number of treatments $k = 4$ and the number of observations $n = 20$. Then, the value of F statistics is
- (A) 7.3
 - (B) 8.3
 - (C) 9.3
 - (D) 19.3

131. is a method of analyzing the tasks involved in completing a given project.

- (A) Graphical method
- (B) CPM
- (C) Big M method
- (D) PERT

132. Which loop executes a sequence of statements multiple times and abbreviates in R code that manages the loop variable?

- (A) for
- (B) while
- (C) do-while
- (D) repeat

133. What will be the output of the following R program?

```
R<- 0:10  
R[2]
```

- (A) 0
- (B) 1
- (C) 2
- (D) 3

134. What will be the output of the following R code?

```
> m <- matrix(nrow = 2, ncol = 3)  
dim(m)
```

- (A) 3 3
- (B) 3 2
- (C) 2 3
- (D) 2 2

135. A random variable that assumes a finite or a countably infinite number of values is called

- (A) Continuous random variable
- (B) Discrete random variable
- (C) Irregular random variable
- (D) Uncertain random variable

136. The covariance of two independent random variable is
- (A) 1
 - (B) 0
 - (C) -1
 - (D) Undefined
137. Consider a random variable with exponential distribution with $\lambda = 1$. Compute the probability for $P(X > 3)$.
- (A) e^{-3}
 - (B) e^{-1}
 - (C) e^{-2}
 - (D) e^{-4}
138. The graph showing the paired points (X_i, Y_i) is called a
- (A) Scatter Diagram
 - (B) Histogram
 - (C) Bar Diagram
 - (D) Pie Diagram
139. The following are percentages of fat found in 5 samples of each of two brands of baby food:
- A: 5.7, 4.5, 6.2, 6.3, 7.3
B: 6.3, 5.7, 5.9, 6.4, 5.1
- Which of the following procedures is appropriate to test the hypothesis of equal average fat content in the two types of ice cream?
- (A) Paired t-test with 5 d.f
 - (B) Two samples t-test with 8 d.f
 - (C) Paired t-test with 4 d.f
 - (D) Two samples t-test with 9 d.f
140. What is the probability of Type-II Error when $\alpha = 0.05$?
- (A) 0.025
 - (B) 0.95
 - (C) 0.05
 - (D) Cannot be determined without more information

141. In the test concerning mean of a normal population t-test is used when
- (A) σ is known and n is large
 - (B) σ is known and n is small
 - (C) σ is unknown and n is large
 - (D) σ is unknown and n is small
142. If $Y = 5X + 10$ and X is $N(10, 25)$, then mean of Y is
- (A) 60
 - (B) 135
 - (C) 70
 - (D) 50
143. Normal Distribution is
- (A) Multi-Modal
 - (B) Bi-Modal
 - (C) Tri-Modal
 - (D) Uni-Modal
144. In one-way ANOVA with total number of observations is 15 with 5 treatments then total degrees of freedom is
- (A) 75
 - (B) 3
 - (C) 10
 - (D) 14
145. What are the factors in a factorial design?
- (A) Independent variables
 - (B) Dependent variables
 - (C) Organismic variables
 - (D) Experimental variables
146. What term refers to the situation where two independent variables have an effect when they are in combination?
- (A) Confounding variable
 - (B) Artifact
 - (C) Dependency
 - (D) Interaction

147. A set of values x_1, x_2, \dots, x_n which satisfies a constraints of the linear programming problems is called
- (A) Optimal solution
 - (B) Feasible solution
 - (C) Objective function
 - (D) Solution
148. If the primal problem is unbounded then the dual problem is
- (A) Feasible
 - (B) Bounded
 - (C) Infeasible
 - (D) Unbounded
149. R allows integration with the procedures written in
- (A) C
 - (B) Ruby
 - (C) Java
 - (D) Basic
150. Which one of the following is **NOT** a basic data type?
- (A) Numeric
 - (B) Character
 - (C) Data frame
 - (D) Integer

FINAL ANSWER KEY

Subject Name: 614 STATISTICS

SI No.	Key								
1	C	31	A	61	B	91	A	121	D
2	B	32	B	62	A	92	D	122	C
3	D	33	B	63	B	93	D	123	A
4	B	34	D	64	B	94	C	124	B
5	A	35	D	65	D	95	C	125	B
6	C	36	B	66	B	96	A	126	B
7	D	37	C	67	A	97	A	127	B
8	D	38	B	68	A	98	A	128	B
9	B	39	A	69	D	99	C	129	A
10	D	40	B	70	B	100	B	130	B
11	B	41	A	71	C	101	C	131	D
12	A	42	D	72	B	102	A	132	D
13	C	43	B	73	B	103	B	133	B
14	B	44	A	74	B	104	B	134	C
15	D	45	B	75	A	105	A	135	B
16	B	46	C	76	B	106	A	136	B
17	B	47	B	77	D	107	C	137	A
18	A	48	C	78	A	108	D	138	A
19	D	49	C	79	C	109	C	139	B
20	B	50	A	80	B	110	C	140	D
21	A	51	D	81	A	111	C	141	D
22	A	52	D	82	C	112	C	142	A
23	B	53	A	83	A	113	D	143	D
24	D	54	B	84	B	114	D	144	D
25	D	55	A	85	D	115	C	145	A
26	C	56	C	86	C	116	B	146	D
27	C	57	B	87	C	117	A	147	B
28	B	58	C	88	D	118	B	148	C
29	C	59	C	89	D	119	C	149	C
30	B	60	C	90	C	120	B	150	C

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